Swami Ramanand Teerth Marathwada University, Nanded.

B.A./B.Sc. Third Year

Syllabus (Mathematics)

Effective from June -2010

B.A. /B.Sc. T.Y. (Mathematics)

Theory Paper-VIII: ANALYSIS .

No. of Periods 120 **Max. Marks: 100**

Unit I : The Riemann Integral: Definitions & existence of the integral. Refinement of partitions. Darboux's Theorem. Condition of integrability. Integrability of the sum & difference of integrable functions. The integral as a limit of sum. Some Integrable functions. Integration & differentiation. The fundamental theorem of calculus Mean Value Theorem.

Unit II : Improper integrals- Introduction. Integration of Unbounded Functions with finite limits of Integration. Comparison Tests. Infinite Range of Integration. Integrand as a product of functions. Abel's Test. Dirichlet's Test.

Unit III : Fourier Series: Trignometrical Series. Some Preliminary Theorems. The Main Theorem. Fourier Series for even & odd functions. Intervals other than [-π,π].
Metric Space: - Definitions and Examples. Open and Closed sets. Convergence and Completeness.
Continuity and Uniform continuity. Compactness. Connectedness

Unit IV : Complex Analysis: Functions of complex variables. Limit, Continuity and differentiability of a function of a complex variable. Analytical function. Cauchy-Riemann Equations in polar form. Orthogonal curves. Harmonic functions. Conjugate function. Conformal transformation. Bi-Linear Transformation.

Text Book : 1) Mathematical Analysis – By S.C. Malik and S. Arora.

Scope **Unit I:** Chapter-9-Art.: 1.1 to 1.3, 2,3,4 (theorem-3 without proof.) 4.1, 5,5.1 (theorem-10 without proof, and other theorems Statement only) 6.1,6.2 (Statements of results only), 7 (theorem-14,15 Statement only), 8,9,9.1,10,10.1
Unit II: Chap 11 Complete.(Art.-5 Statement of theorem only)
Unit III : Chap. 14:- Art-4(Statement of theorem Only/ Result without proof).
Chap 19: Art-1,2,(2.7 Statement / Result without proof), 4,4.1,5(Statement of theorem 21Only), 6.(Statement of theorem 40,41,42 Only).

2) Advanced Engineering Mathematics- By H. K. Dass.
Unit IV: Chap.- 7-Art: 7.1 to 7.27.

Reference Books

1	Methods of Real Analysis	:	By Richard R.Goldberg.
2	An introduction to Real Analysis	:	By P.K. Jain and S.K. Kaushik
3	Metric Spaces	:	By. E.T. Copson
4	Mathematical Analysis 1 (Metric spaces)	:	By J.N. Sharma
5	Functions of one Complex variable	:	By John B.Conway
6	Introduction to Topology and Modern Analysis	:	By B.F. Simmons

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Theory Paper-IX-ABSTRACT ALGEBRA.

No. of Periods 120 **Max. Marks: 100**

Unit I : Group-Automorphisms. Inner Automorphisms. Conjugacy Relation. Counting principal, Class equation of a finite group. Sylow's Theorem. P-Sylow subgroup, Direct Products.

Unit II : Ring Theory-Ring homomorphism. Ideals & Quotient Rings. More ideals and Quotient Rings. Field of Quotients of an integral domain. Euclidean Rings. Polynomial rings, Polynomial over a rational field, Polynomial ring over Commutative rings.

Unit III : Vectors Spaces-Elementary basic concepts. Linear Independence and Bases. Dual spaces. inner Product spaces(definitions and examples only)

Unit IV : Linear Transformations-The Algebra of Linear Transformations. Characteristic roots. Matrices.

Text Book : Topics in Algebra. By I. N. Herstein.

Unit-I- : **Chapter-2-Art** 2.8,2.11,2.12 (Second proof of Theorem 2.12.1 and other results without proof) 2.13.

Unit-II : **Chapter-3-Art** 3.8, 3.9, 3.10, 3.11(delete particular Eucliden Ring)

Unit-III : **Chapter-4-Art** 4.1,4.2,4.3, 4.4 (Delete Dual spaces)

Unit-IV : **Chapter-6-Art** 6.1, 6.2, 6.3

Reference Books

1 Lectures in Abstract Algebra : By N. Jacobson

2 Basic Abstract Algebra : By P. Bhattacharya S.K. Jain

3 Linear Algebra : By K. Hoffman, K. Kunze

4 A First Course in Abstract Algebra : By Fraleigh

5 Abstract Algebra: By V.K. Khanna, S.K. Bhambri

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Theory Paper-X (A): Numerical Analysis & Topology

No. of Periods 120 **Max. Marks: 100**

Unit I : Differences, Operators, Interpolation with equal intervals: Operators E,∇,Δ,D Newton Gregory Formula for forward and backward interpolation. Interpolation for unequal intervals of the arguments: Divided differences. Newton's, Lagrange's formulae for unequal intervals.

Unit II : Central Differences Interpolation formulae: Gauss, Bessel and Stirling formulae, Numerical differentiation. Approximate expression for the derivative of a function. Numerical quadrature: General formula, Trapezoidal, Simpson's one-third and three-eight rules.

Unit III : Numerical solution of O.D.E.: Euler's Method, Euler's modified Method, Picard's Method. Method of starting the solution by Taylor's series, Milne's series.

Unit IV : **Topology** Topological spaces and continuous functions: Definition and examples, Finer Topology. Topology generated by basis. Standard Topology. Sub basis. Product Topology. Projection Mapping. Subspace Topology
Closed Sets, Limit points, Closure and interior of a set, Hausdroff space. Continuous functions. Homeomorphisms

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Text Book : 1. Finite differences and Numerical Analysis. By H.C. Saxena. S. Chand and Co. New Delhi.
Chapter-1:Art. 1.1, 1.2, 1.3, 1.5.1, 1.6, 1.6, 1.6.1, 1.6.2, 1.7, 1.8, 1.8.1, 1.8.2, 1.8.3.
Chapt-2:Art. 2.1, 2.1.1, 2.2 (Theorem 1 and 2 only) 2.3, 2.4, 2.4.1.
Chapt-3: 3.1, 3.2, 3.3, 3.4, 3.5.
Chapt- 5: 5.1, 5.2
Chapt-6:6.1, 6.2, 6.3, 6.3.2, 6.3.3.
Chapt-15:15.1, 15.2, 15.2.1, 15.2.2, 15.2.3, 15.2.4, (a), (b)
2. Topology a First Course
James R. Munkres. Prentice-hall of India, New Delhi. 1983. Chapter-2.:2.1, 2.2, 2.4, 2.5, 2.6, 2.7.

Reference Books

1. Numerical Mathematical Analysis By J.B. Scarborugh.
2. Introductory Methods of Numerical Analysis By S.S. Sastry.

3 Introduction to Numerical Analysis	By C.E. Froberg
4. General Topology	By S. Willard
5. Introduction to Topology	By K.D. Joshi

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Theory Paper-X (B):<u>ORDINARY DIFFERENTIAL EQUATIONS</u>.

No. of Periods 120 **Max. Marks: 100**

Unit I **:** Linear Equations of the first order: Differential Equations, Linear equations of first order, The general linear equation of the first order. Linear equations with constant coefficients. The second order homogeneous equation, the non-homogeneous equation of order two.

Unit II : Linear equations with constant coefficients: The homogeneous equation of order n, The non-homogeneous equation of order n. Linear Equations with variable coefficients: initial value problems for homogeneous equation, solutions of the homogeneous equation.

Unit III : Liner equations with variable coefficients: The Wronskian and Linear independence, Reduction of order, The non-homogeneous equation, The Euler equation.

Unit V : Existence and Uniqueness of solutions to first order equations: The method of successive approximations, the Lipschitz condition, Approximation to, and uniqueness of, solutions.

Text Book : An introduction to ordinary Differential Equations:- Earl A. Codington
Prentice-Hall of India Private Ltd., New Delhi.

Scope:

Unit I : Chapter-1: Articles 1 to 7 (pages 33-48)
Chapter-2: Articles 1 to 6 (pages 49-70)

Unit II : Chapter-2: Articles 7,8,10,11 (pages 71-79, 84-93)
Chapter-3: Articles 1,2,3 (pages 103-110)

Unit III : Chapter-3: Articles 4 to 8 (pages 111-137)
Chapter-3: Articles 1,2 (pages 143-150)

Unit IV : Chapter-5: Articles 4 to 6,8 (pages 216,222-226)

Reference Books :1: Ordinary Differential Equations By W.T. Reid.
2: Theory & Problems of Differential Equations By Frank Ayres.

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Theory Paper-X (C)

NUMBER THEORY AND DISCRETE MATEMATICS

No. of Periods 120 **Max. Marks: 100**

Unit I **:** Divisibility: Introduction, Divisibility, Primes congruence, solution of congruences, congruence of degree one, The Euler's phi function.

Unit II : Number theory from algebraic view point, Multiplicative group, ring and fields, Quadratic reciprocity, The Jacobi symbol. Greatest integer function, Arithmetic functions, Mobius inversion formula, Multiplication of arithmetical functions.

Discrete Mathematics

Unit III : Graphs, planar graphs, Multigraphs and weighted graphs, Eulerian paths and circuits, Hamilton paths and circuits, Trees, Rooted trees, Path lengths, Binary search trees, Spanning trees and Cutsets, minimum spaning trees.

Unit IV : Boolean algebra, Lattices, Algebraic system, principle of duality, basic properties of Distributive and complimented lattices, Boolean lattices, Boo lean algebra, uniqueness of finite Boolean algebras, Boolean functions and Boolean expressions.

Text Book : 1. An introduction to the Theory of numbers
By Ivan Niven & Hebert S.Z.

Unit I : Chapter-1: Complete.
Chapter-2: Articles 2.1 to 2.4

Unit II : Chapter-2: Articles 2.10, 2.11
Chapter-3: Complete
Chapter-4: Articles 4.1 to 4.4
2) Elements of Discrete Mathematics –C.L.Liu.

Unit III : Chapter-5: Articles 5.1 to 5.7
Chapter-6: Articles 6.1 to 6.7

Unit IV : Chapter-12: Articles 12.1 to 12.7

Reference Books

1. Introduction to Number Theory : By W.W. Adams & L.J. Goldstein
2. A Text Book of Graph Theory : By John Clerk & D.A. Holton
3. science Discrete Mathematical Structures with Application to Computer : By J.P. Trembley & R. Manohar

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(PRACTICAL) -Paper-XI

NUMBER THEORY AND DISCRETE MATEMATICS

Note:

1. The practical paper is only for B.Sc. Students.
2. For a batch of 20 students, 2 periods per week, will be the work load.
3. A Record book, consisting of at least 50% of the practical given below be maintained by each students.
4. The theory part required for the practical be explained to the students, by the teacher concerned, from the text and reference books.

PRACTICAL BASED ON THEORY PAPER VIII:ANALYSIS:

1. If f is continuous and non-negative on [a,b] then show that.

   $\int_a^b f(x)\, dx^3 \geq 0$

2. Show that the function.

   f (x) = x,   when x is rational

   = -x,   when x is irrational

   is not integrabel over [a,b] but | f | is integrable.

3. Examine the convergence of

   $\int_0^\infty x^{m-1}$ e^{-x} dx.

4. Obtain the Fourier series in [-π,π] for the function.

   f (x) = x, if -π≤ x ≤0

   = 2x, if 0≤ x ≤ π

5. Prove that the function d^*: R^n x R^n → R defined by

   d^* (x,y) = max $|x_2-y_2|$,

$\forall x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n) \in R^n$.

$1 \leq i \leq n$,

is a metric on R^n.

6. Prove that Hilbert space is a separable metric space.
7. Let X be an infinite set with the discrete metric, show that (X,d) is not compact.
8. If {An} is a sequences of connected subsets of a metric space x, each of which intersects its successor, i.e., $A_n \cap A_{n+1} \neq \underline{V}$. $n \in N$, $\bigcup_{n=1}^{\infty} A_n$,then show that An is connected.
9. Find the conjugate function of $u = \frac{1}{2}\log (x^2+y^2)$
10. Determine the analytic function whose real part is e^{2x} (x.cos2y-y.sin2y)

PRACTICAL BASED ON THEORY PAPER :VIII ABSTRACT ALGEBRA:

11. Show that a group of order $11^2 \times 13^2$ is abelian.
12. Prove that group of order 15 is cyclic.
13. If U is an ideal of a ring R and r (U) = {x $\in$ R / xu = 0 for all u $\in$ U), prove that r (U) is an ideal of R.
14. Let R be a ring with unit element. Using its elements we define a ring R by a +b = a+b+1 & a b = a b + a + b, where a, b $\in$ R. Prove that R is a ring.
15. Let J be the ring of integers, p a prime number, and (p) the ideal of J consisting of all multiples of p, prove that J/(p) is isomorphic to J_p , the ring of integers mod p.
16. If V is finite-dimensional vector spaces and W is a subspace of V, prove that there is a subspace W_1 of V such that $V = W \oplus W_1$.
17. If $\{W_1, \ldots W_m\}$ is an orthonormal set in V. prove that $\sum |(w_i.v)|^2 \leq \|v\|^2$ for any $v \in V$

18. In a vector space V prove the parallelogram law;

$|| u+v||^2+||u-v||^2 = 2(||u||^2+||v||^2)$

19. For T A (V), prove that range of T, VT = {v T | v∈V} and U = {v∈T | v T=0} are subspaces of V.

20. Show that r (T) + n(T) = dim V , Where n (T) is dimension of U = {v∈T| v T=0}.

<u>PRACTICALS BASED ON THEORY PAPER X (A) / X (B)/ X (C)</u> .

Choose either X(A) or X(B) or X(C):

<u>X(A):NUMBERCAL ANALYSIS:</u>

21. Show that if n is a positive integer, then

$\Delta^n \sin(a+bx) = (2\sin\frac{b}{2})^n x \sin[a+bx+\frac{n}{2}(b+\pi]$,1 is the interval of differencing.

22. Show that

i) $\nabla f(x)g(x) = f(x)\nabla g(x) + g(x-1)\nabla f(x)$

ii) $\nabla f(x)/g(x) = f(x)\nabla g(x) - g(x-1)\nabla f(x).\ g(x-1)g(x)b$

23. Show that

i) $\delta[f(x)g(x)] = \mu\ f(x)\delta\ g(x) + \mu\ g(x)\delta\ f(x).$

ii) $\delta\ [\frac{f(x)}{g(x)}] = \frac{\mu g(x)\delta\ f(x) - \mu\ \mathrm{f}(x)\delta\ \mathrm{g}(x)}{g(x-\frac{1}{2})g(x+\frac{1}{2})}.$

24. Show that

i) $\mu\ \mathrm{f}(x)g(x) = \mu\ f(x)\mu\ g(x) + \frac{1}{4}\delta f(x)\delta\ \mathrm{g}(x).$

ii) $\mu[\frac{\mathrm{f}(x)}{\mathrm{g(x)}}] = \frac{\mu\ f(x)\mu\ g(x) - \frac{1}{4}\delta f(x)\delta\ \mathrm{g}(x)}{g(x-\frac{1}{2})g(x+\frac{1}{2})}.$

25. A river is 80 ft. wide, The depth d(in feet of the river at a distance x from one bank is given by the following table:

X:	0	10	20	30	40	50	60	70	80
D:	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross section of the river, using (i) five suitable points, (ii) all the points of the data.

26. A rocket is launched from the ground, its acceleration is registered during the first 80 seconds and is given as:

t(sec):	0	10	20	30	40	50	60	70	80
a(m/sec^2):	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

Find the velocity and height of the rocket at t=80

X(B):Topology:

27. If x = {a, b, c}, let

$J_1 = \phi, x, \{a\}, \{a, b\}\}$ and $J_2 = \{\phi, x, \{a\}, \{b, c\}\}$.

Find the smallest topology containing J_1 and J_2, and the largest topology contained in J_1 and J_2.

28. Apply Lemma 2.3 to show that countable collection is a basis that

$B_1' = \{(a,b) / a < b, a \text{ and } b \text{ rational}\}$

is a basis that generates the standard topology on R.

$B_2' = \{(a,b) / a < b, a \text{ and } b \text{ rational}\}$

is a basis that generates a topology different from the lower limit topology on R.

29. A map f :X→ Y is said to be an open map if for every open set U of X, the set f (U) is open in Y. Show that π: X x Y → X and π: X x Y → Y are open maps.

30. Let $A \subset X$ and $B \subset Y$. Show that in the space X x Y, $\overline{A \times B} = \overline{A} \times \overline{B}$

X (B) ORDINARY DIFFERENCTIAL EQUATIOSN:

21. Consider the equation $y' = ky$ on $-\infty < x < \infty$, where K is some constant, then show that, if ϕ is any solution, and $\psi(x) = \phi(x)e^{-kx}$, then, $\psi(x) = c$, where c is a constant.

22. Consider the Equation

$$Ly' + Ry = E \sin \omega x$$

Where L, R, E, ω are positive constants, then computer the solution satisfying ϕ (o) =0.

23. Consider the equation

$$Ly'' + Ry + \frac{1}{C} y = 0$$

Where L, R and C, ω are positive constants (L is not a differential operator, then compute all the solutions for the three cases.

$i)\frac{R^L}{L^2} - \frac{4}{LC} > 0 \qquad ii)\frac{R^L}{L^2} - \frac{4}{LC} = 0 \qquad iii)\frac{R^L}{L^2} - \frac{4}{LC} < 0$

24 Suppose that ϕ and ψ are two solutions of the constant coefficient equation

$$L(y) = y'' + ay' + a_2 y = 0$$

On a finite interval I include a point x_0. Let

$\phi(x_0) = a_1$, $\phi(x_0) = b_1$

$\psi(x_0) = a_2$, $\psi(x_0) = b_2$

and suppose $(a_1-a_2)^2 + (b_1-b_2)^2 = \in^2$, and if $x = \phi - \psi$ show that $L(\chi) = 0$, and $\chi(x_0) = a_1 - a_2$, $\chi(x_0) = b_1 - b_2$.

25 a) Computer the Wronskian of four linearly independent solutions of the equation $y^{(4)}+16y = 0$

b) Computer the solution of the equation which satisfies

$$\phi(0) = 1, \phi(0) = 0, \phi''(0) = 0, \phi'''(0) = 0$$

26. Find two Linearly independent solutions of the equation

$(3x-1)^2 y'' + (9x-3)y' - 9y = 0$ for $x > \frac{1}{3}$.

27. One solution of L(y) = $y'' + y = 0$, for x $>$0 is ϕ(x) = x^2. Show that there is another solution ψ of the form ψ = u ϕ, where u is same function.

28. Two solutions of $x^2 y'' - 3xy' = 0, \quad (x > 0)$, are ϕ_1(x) = x , ϕ_2(x) = x^3.Use this information to find a third independent solution.

29. For the problems $y'' = x^2 + y', y(0) = 0$ and $y' = 1 + xy, y(0) = 1,$ computer the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3.$

30. Show that the functions f given by

F(x , y) = $y^{1/2}$, does not satisfy a Lipschitz condition on

R: $|x| \leq 1, 0 \leq \; y \leq 1$.

X(C):NUMBER THEORY:

21. Solve: $x^2 + x + 7^o \equiv 0$ (Mod 27).

22. Solve: $x^3 + 4x + 8 \equiv 0$ (Mod 15).

23. Solve: 3x-6y + 5z = 11, 6x + 48y-78z = 5.

24. Find all solution in positive integers of the equation 5x + 3y = 52.

25. Find all primitive solution of $x^2+y^2 = z^2$ having o < z < 30.

26. By induction hypothesis show that

$1^n + 2^3 + 3^3$ ………..+ $n^3 = (1+2+3+\ldots+n)^3$.

27. By induction hypothesis show that

$2^n > n^3$ for $n \geq 10$

28. Let A,B, C be any arbitrary sets, Then show that

(A-B)-C=(A-C)-(B-C)

29. For any sets A,B,C.D show that

$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

30. For any two sets A_1 and A_2, show that

$(A_1 \cup A_2) = n(A_1) + n(A_2) - n(A_1 \cap A_2)$.